

From confinement to new states of dense QCD matter

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Abstract

Transitions between centre sectors are related to confinement in pure Yang-Mills theories. We study the impact of these transitions in QCD-like theories for which centre symmetry is *explicitly* broken by the presence of matter. For low temperatures, we provide numerical evidence that centre transitions do occur with matter merely providing a bias towards the trivial centre sector until centre symmetry is *spontaneously* broken at high temperatures. The phenomenological consequences of these transitions for dense hadron matter are illustrated in the Schwinger model and an SU(3) effective quark theory: centre dressed quarks undergo condensation due to Bose-type statistics forming a hitherto unknown state of dense but cold quark matter.

1 Introduction

Pure Yang-Mills theories feature a colour-confinement phase at low temperatures separated by a phase transition from the gluon plasma phase at high temperatures. This phase structure is well understood by virtue of the centre symmetry which is realised at low but spontaneously broken at high temperatures [1, 2]. The (temporal) Polyakov line transforms homogeneously under a centre transformation of the gluon fields, and, thus, its expectation value serves as an order parameter for confinement.

The situation is less clear cut in QCD-like theories where matter transforming under the *fundamental* representation of the gauge group is coupled to the $SU(N_c)$ gluon fields. Here, centre symmetry is explicitly broken and, strictly speaking, quark confinement is absent since the colour-electric flux tube between a static quark antiquark pair can break by means of the creation of a dynamical (light) quark antiquark pair. The string breaking distance r_B can be estimated by

$$r_B \approx 2M/\sigma,$$

where M is the (constituent) mass of the dynamical matter, and σ is string tension of the corresponding pure Yang-Mills theory. Thus, in the case for heavy matter, QCD-like theories at intermediate scales, i.e., $1/\sqrt{\sigma} < r < r_B$, are well described by their pure Yang-Mills counter part. The Polyakov line is almost disordered by the approximate centre symmetry, and its expectation value is only non-zero due to bias towards the trivial centre sector by the matter fields.

In certain gauges and using lattice gauge simulations, confinement has been successfully attributed to certain gluonic degrees of freedom. For later use, we would like to mention the centre vortex picture (see [3] for a review): vortices experience an intrinsically fine-tuned balance between energy and

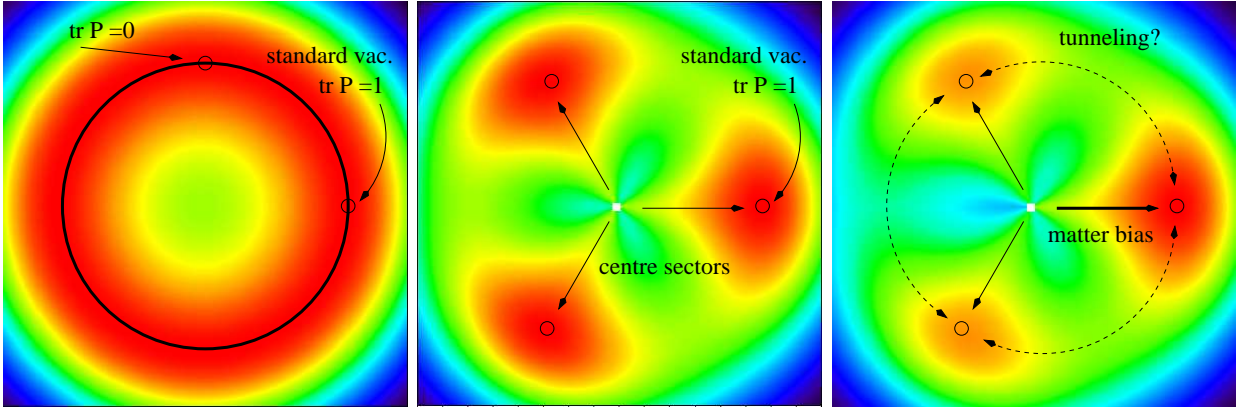


Figure 1: Sketch of the *classical action* as function of the Yang-Mills configuration space (left); *effective action* of pure Yang-Mills theory (middle); *effective action* of QCD-like theories, i.e., YM theory with dynamical matter, (right).

entropy [4] implying that their intrinsic energy scale is set by the physical confinement scale [5] (rather than the UV regulator of the theory). It was argued recently that vortex configurations are the image (in the maximal centre gauge) of smooth instanton-like configurations (which do confine) thus explaining the intrinsic fine-tuning between vortex energy and entropy [6]. Vortex percolation is directly related to the disorder of the centre symmetry, and vortex de-percolation explains the finite temperature de-confinement transition to the hot gluon plasma phase [7, 8, 9].

The approximate centre symmetry in QCD-like theories has strong phenomenological consequences if dense matter is considered: At least for theories with an even number of colours, quarks which are subjected to certain gluonic background fields acquire Bose statistic and can undergo condensation [10], the so-called *Fermi-Einstein condensation* [11]. Quite recently, this mechanism has been thoroughly studied using the exactly solvable Schwinger model and resorting to effective quark models. It was found [12] that the mechanism extends its scope to theories with an odd number of colours (though pressure needs to be applied to the dense medium) and it was conjectured that a new state of cold, but dense matter might exist in the hadronic phase of QCD for which Fermi-Einstein condensation is realised.

In this paper, we review the properties of the approximate centre symmetry and the line of arguments leading to Fermi-Einstein condensation. We review the exact result from the Schwinger-model in [12] and discuss the possibilities to turn the SU(3) quark model of [12] from minimal to realistic.

2 Fermi-Einstein condensation

2.1 Yang-Mills moduli and vacuum structure of QCD-like theories

Let us firstly discuss pure Yang-Mills theory on a toroidal space-time. We will call an *empty vacuum* a gauge field configuration $A_\mu(x)$ for which any holonomy along a contractible loop \mathcal{C} is the unit element:

$$W_{\mathcal{C}} = \mathcal{P} \exp \left\{ i \int_{\mathcal{C}} A_\mu(x) dx_\mu \right\} = 1, \quad \mathcal{P} : \text{path ordering.} \quad (1)$$

How many of these configurations are there? Obviously, $A_\mu(x) = 0$ is an example, and since $W_{\mathcal{C}}$ is an element of the adjoint representation, all gauge equivalents of this configuration are also empty

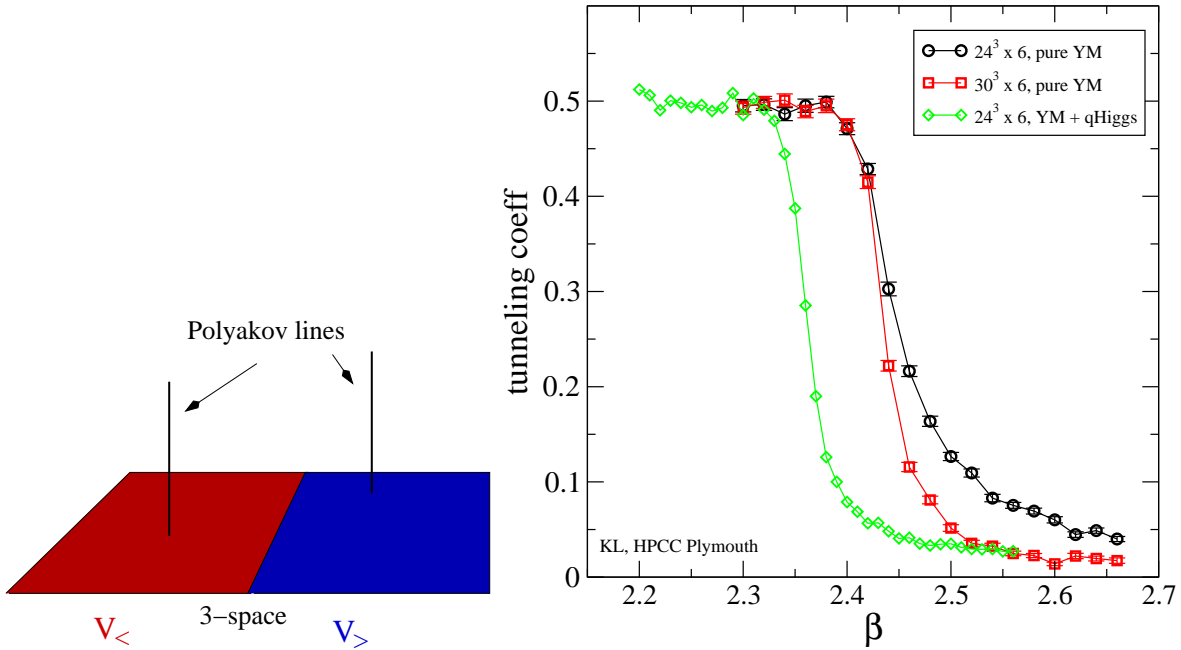


Figure 2: Left: Illustration of the “Litmus paper” for centre sector transitions (left). Right: The tunneling coefficient p as a function of the Wilson β for the pure SU(2) gauge theory (black and red) and for the SU(2) Higgs theory (plot from [12]).

vacua. The more interesting question is then: how many *gauge inequivalent* field configurations are there which all satisfy (1). It turns out [13, 14, 15] that at least a smooth $U(1)^{4(N_c-1)}$ manifold of gauge inequivalent configurations exists (see e.g. [12]). For a map of this manifold, we define the Polyakov line in μ -direction by

$$P_{(\mu)}(x) = \mathcal{P} \exp \left\{ i \int_{\ell} A_{\mu}(x) dx_{\mu} \right\} \quad (\text{no sum over } \mu), \quad (2)$$

where ℓ is the straight line starting at x winding through the torus in μ direction. The empty-vacuum configurations are then labeled by constant Polyakov lines which take values in the Cartan subgroup of the gauge group.

$$P_{(\mu)}(x) = P_{(\mu)} \in \text{Cartan subgroup}, \quad \mu = 1 \dots 4.$$

The situation is illustrated in figure 1, left panel, which is a cartoon of the classical action as a function of the gauge invariant configuration space (all gauge equivalent configurations are identified with one point of the canvas). The manifold of least action is indicated by the circle. Also indicated is the standard perturbative vacuum $A_{\mu}(x) = 0$ as well as an empty vacuum configuration with a vanishing trace of the Polyakov line. One can show (see e.g. [12]) that the Polyakov line correlator, calculated with any of the empty vacuum configurations \mathcal{E} ,

$$\left[\text{tr } P_{(\mu)}(x) \text{tr } P_{(\mu)}(y) \right]_{\mathcal{E}} = \text{constant},$$

and, hence, does not support a non-trivial quark antiquark potential. Below, however, we will argue that a patch configuration which consists of regions in space-time with different empty-vacuum-configurations, is well suited to describe confinement.

At quantum level, the Coleman effective action as a function of the so-called classical gluon field needs to be considered, and the effective action significantly differs from the classical action for a

strongly coupled theory. For the pure SU(3) Yang-Mills theory, such an effective action is sketched in figure 1 (picture in the middle). The “flat directions” of the classical action are lifted by the quantum fluctuations. Note, however, that the discrete Z_3 centre transformations ($N_c = 3$ below), e.g.,

$$A_\mu(x) \rightarrow A_\mu^z(x) = A_\mu(x) + \frac{2\pi n}{L} H \delta_{\mu 0}, \quad 0 \leq n < N_c, \quad H = \text{diag}(1, \dots, 1, 1 - N_c) / N_c, \quad (3)$$

where L denotes the size of the torus in any direction, is still an invariance of the effective action. If $[P]$ denotes the space-time average of a particular configuration, we can map each configuration to a centre sector n by

$$c([P]) = n : \quad \text{Re}(\text{tr } P^\dagger Z_n) \rightarrow \max, \quad Z_n = \exp\{i 2\pi n H\} \quad (4)$$

In figure 1, picture in the middle, the three centre sectors mark the the three global minima of the effective action. The pure Yang-Mills configuration space bears enough entropy that transitions between these centre sectors do still occur even at large volumes. These transitions are then associated with colour confinement. Only at high temperatures, the centre symmetry *spontaneously breaks*, centre sector transitions cease to exist and the deconfined gluon plasma phase is adopted.

Pure Yang-Mills theories differ from QCD-like theories by the presence of dynamical matter which transforms under the fundamental representation of the group. In QCD-like theories, the Z_N symmetry is *explicitly* broken by the matter-gluon interactions. The effective potential (see figure 1 right panel) now possesses a unique global minimum which belongs to the trivial centre sector. At least for heavy matter, the degeneracy is only lifted by a small amount, and the questions rises whether transitions between centre sectors do still take place in such theories. A Litmus paper which is sensitive to sector transitions was constructed in [12]: Dividing the spatial volume into two regions $V_>$ and $V_<$ of equal size, we calculate the spatial average of the Polyakov line, $P_>$ and $P_<$, over each of these regions (see figure 2). Each region of the lattice is then mapped to a centre sector by $c(P_{</>})$ (4). The “Litmus paper” is then given by the probability p that both regions belong to different sectors. If for an $SU(N_c)$ QCD-like theory, the centre sectors are attained at random in both regions, we find $p = 1 - 1/N_c$. By contrast, if sector transitions have stopped at high temperatures, both regions belong to the *same* centre sector implying $p = 0$. The result for the so-called tunneling coefficient p for the pure SU(2) gauge theory as a function of the Wilson β coefficient is shown in figure 2, right panel. The finite temperature deconfinement phase transition is clearly visible as the transition from $p \approx 1/2$ to $p \approx 0$. We find qualitatively the same behaviour for the SU(2) Higgs theory though the transition occurs at a smaller value of β indicating a slip of the critical temperature to smaller values. Most importantly, $p \approx 1/2$ in the string-breaking phase at small β indicating that centre sector transitions do frequently occur.

2.2 Impact of centre-sector transitions

Let us now explore the phenomenological impact of the centre-sector transitions if dynamical quarks are included. To start with quarks satisfy anti-periodic boundary conditions. If $A_\mu(x)$ is a generic gluonic background field, we denote the quark contribution to the QCD partition function by the quark determinant $\text{Det}_{\text{AP}}[A_\mu]$ (where AP is indicating the anti-periodic boundary conditions for the quarks). If centre-sector transitions do occur, the gauge fields $A_\mu^z(x)$ in (3) are also a generic part of the gluonic ensemble average. Using a change of variables, the so-called Roberge-Weisz transformation (see [10] for details, one shows that the identity for the probabilistic measure

$$\text{Det}_{\text{AP}}[A_\mu^z] \exp\{-S_{\text{YM}}[A_\mu^z]\} = \text{Det}_{(n)}[A_\mu] \exp\{-S_{\text{YM}}[A_\mu]\},$$

where the determinant on the right hand side is obtained by integrating quark fields which satisfy Z_n -periodic boundary conditions:

$$q(x + L e_0) = (-1)^{Z_n} q(x), \quad Z_n \text{ in (4)}.$$

Hence, the partition function of QCD (or the QCD-like) theory can be written as

$$\int \mathcal{D}A_\mu \text{Det}_{\text{AP}}[A_\mu] \exp\{-S_{\text{YM}}[A_\mu]\} = \int \mathcal{D}A_\mu^{(0)} \left(\sum_n \text{Det}_{(n)}[A_\mu^{(0)}] \right) \exp\{-S_{\text{YM}}[A_\mu^{(0)}]\}, \quad (5)$$

where the gauge fields $A_\mu^{(0)}$ are only drawn from the trivial centre sector.

The phenomenological consequences of (5) for the theory at *finite densities* was spelled out in [10]. For an even number of colours N_c , $Z_{N_c/2} = (-1)$ is an element of the group implying the sum over the determinants in (5) includes a quark determinant where the quarks obey periodic boundary conditions. This implies that the quark fields possess Bose-statistics, and the quark operator possesses a mode with vanishing Matsubara frequency. If the baryon chemical potential approaches the quark mass gap, the $n = N_c/2$ contribution in N_c dominates, and, at low temperatures, the quark free energy develops a logarithmic singularity reminiscent that of the Bose-Einstein condensation. This has been called *Fermi-Einstein condensation* (FEC). We stress here that for FEC to occur, the centre-sector transitions still need to take place since otherwise the sum in (5) would collapse to the contribution $n = N_c$ from the trivial centre sector due to the *spontaneous breakdown of centre symmetry*. Hence, FEC only applies to the confinement phase for which quarks cannot be considered as asymptotic states, and no contradiction to the spin-statistic theorem occurs. In this case, the observable degrees of freedom are hadrons while quarks can be viewed as auxiliary fields (as e.g. ghosts fields are for the gluonic sector if we choose to work in a gauge fixed environment).

FEC has been studied at great length [12] in the Schwinger-model at finite densities, which can be solved exactly [16, 17, 18]. It was found that the centre-sector transitions do take place even under extreme conditions and imply that the partition function becomes independent of the chemical potential thus solving the Silver-Blaze problem in this model.

3 Cold, but dense SU(3) quark matter

So far, FEC is restricted to QCD-like theories with an even number of colours. The question arises whether there is any trace of FEC left in theories with an *odd* number of colours and, most importantly, in QCD for $N_c = 3$. Even after gauge fixing, the left hand side of (5) would hardly admit a perturbative expansion with respect to small gauge fields since centre copies of $A_\mu = 0$ are equally relevant and correspond to large gauge fields. On the other hand, these “large configurations” have been taken into account in the right hand side by the centre sector sum, and an expansion with respect to $A_\mu^{(0)}$ might be perfectly viable. For the motivation of our quark model, we take this reasoning to the extreme and set $A_\mu^{(0)} = 0$. This model is unrealistic since it does not produce a confining scale from the gluonic sector. It, however, guarantees the absence of coloured state from the partition function in the confinement phase [12].

The partition function of our model is given by

$$Z_Q = \sum_{n=1}^{N_c} \int \mathcal{D}q \mathcal{D}\bar{q} \exp\left\{\bar{q}(i\cancel{\partial} + (A_0^{(n)} + i\mu)\gamma^0 + im)q\right\} = \sum_{n=1}^{N_c} e^{\Gamma^{(n)}}, \quad (6)$$

$$\Gamma^{(n)} = \ln \det(i\cancel{\partial} + (A_0^{(n)} + i\mu)\gamma_0 + im), \quad A_0^{(n)} = 2\pi n T H \quad (7)$$

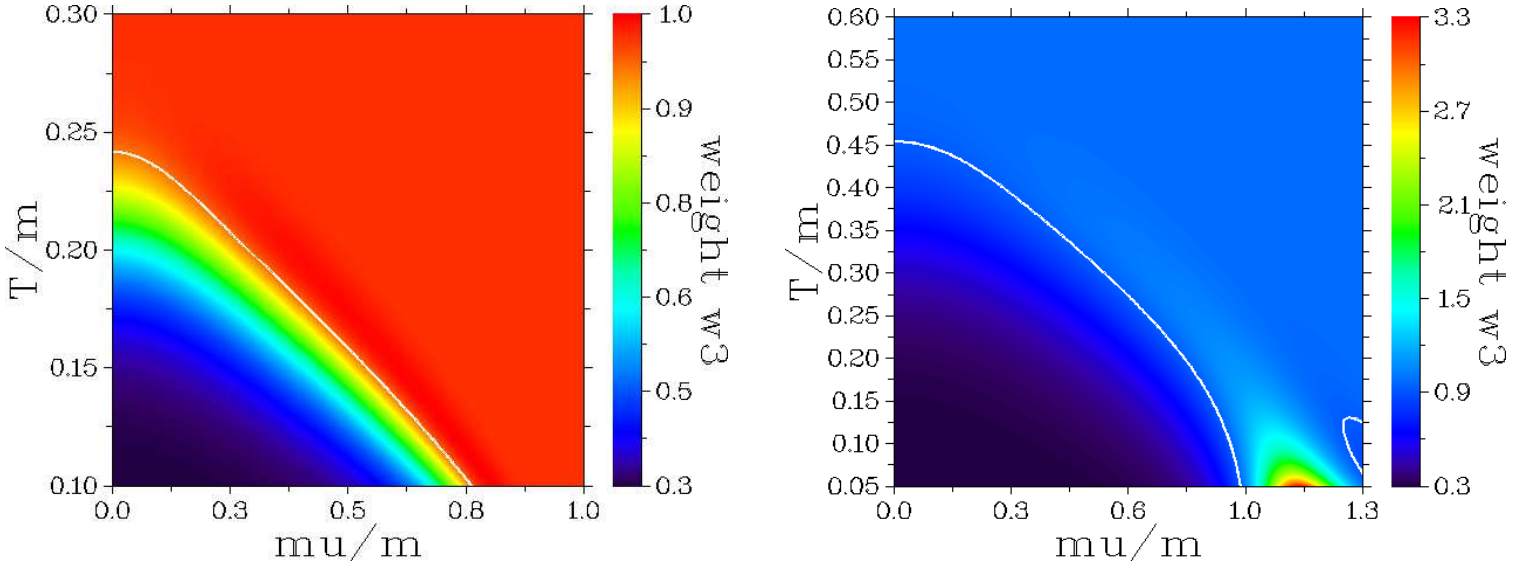


Figure 3: The phase diagram of the SU(3) quark model for a large ($mL = 15$, left panel) and a small ($mL = 5$, right panel); note the different scale in the colour coding; plots from [12].

where H was defined in (3), T is the temperature, m is the quark mass and μ is quark chemical potential. The model can be solved exactly. We here focus on the baryon number which can be written as

$$Q_B = T \frac{\partial \ln Z_Q}{\partial \mu} = \sum_n \omega_n \sum_p \left[\frac{z_n^*}{e^{\beta(E(p)-\mu)} + z_n^*} - \frac{z_n}{e^{\beta(E(p)+\mu)} + z_n} \right], \quad (8)$$

where z_n are the centre phases and where the centre sector weights ω_n are given by

$$z_n = \exp \left\{ i \frac{2\pi}{3} n \right\}, \quad \omega_n = \frac{\exp \{ \Gamma_{\text{den}}^{(n)} \}}{\sum_n \exp \{ \Gamma_{\text{den}}^{(n)} \}}. \quad (9)$$

The weights ω_n can be easily calculated within the model (see [12] for details).

For vanishing chemical potential $\mu = 0$, the weights ω_n are real, positive and smaller (or equal) 1. For this reason, they can be interpreted as the probability with which each of the centre sectors n contribute to the baryon number. It turns out that for small temperatures ($T \ll 0.2m$), the centre sectors roughly contribute with equal weights, $\omega_1 \approx \omega_2 \approx \omega_3 \approx 1/3$, while the centre-symmetry also breaks spontaneously for $T \gtrsim 0.2m$. In the latter case, we find $\omega_1 \approx \omega_2 \approx 0$, $\omega_3 \approx 1$ and our model merges with the standard Fermi-gas model.

For non-vanishing chemical potential $\mu \neq 0$, the weights ω_n are no longer real and cannot be interpreted as sector probabilities anymore. Nevertheless, we still find almost an equal distribution for $\omega_1 \dots \omega_3$ in the hadronic phase, while $|\omega_3| \approx 1$ holds under extreme conditions. In this sense, we can use $|\omega_3|$ to trace out the phase diagram as a function of the chemical potential μ and the temperature T . Our findings [12] are summarised in figure 3. For large spatial volumes $mL > 15$, figure 3, left panel, we find a hadronic phase separated from a deconfinement phase under extreme conditions. Under pressure, i.e., for small spatial volumes $mL < 5$, we find a region of the phase diagram at low temperatures and intermediate values for which $|\omega_3| \gg 1$ holds. In this region, the quark free energy logarithmically diverges, and we find an excess of the baryon number if compared to the standard Fermi gas model for same temperature and chemical potential. From here, we conclude that, at least under pressure, the

new state of cold, but dense matter might also be realised in QCD-like theories with an odd number of colours.

Let us finally comment on the possibilities to render the above SU(3) quark model more realistic: (i) In a phenomenological oriented approach, one might consider an NJL-type four fermion interaction besides of the average over the centre-sector background fields. Here, it would be interesting to see whether a spontaneous breakdown of chiral symmetry also implies a spontaneously broken centre symmetry and hence deconfinement. (ii) In a more fundamentally oriented approach, one notices that the prototypes of centre sector gauge fields configurations are homogeneously stretching throughout space-time. This is a state of low energies but with little entropy. Breaking up the gauge fields into patches of different centre-sector fields significantly increases the entropy and costs energy proportional to the interfaces between the patches. The interfaces can be identified with centre-vortices, and it was observed e.g. in [19] that the vortex interface energy vanishes in the confinement phase leading to percolating vortices. Here, the so-called planar vortex density sets the confinement scale [5]. Hence, supplementing quarks to a SU(3) vortex model would be natural extension of the SU(3) Fermi gas model.

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